

History of Flight Vehicle Structures 1903–1990

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In this paper the authors present a survey of the major developments in structure technology that have influenced modern aircraft design. They also offer their perspectives on the key materials and concepts that drive air vehicle structural design. The authors discuss the focus of research and development structural development resources and address the reasons for this focus.

Introduction

IN spite of the often made claims of revolutionary design in military airframes, construction has historically been very evolutionary in nature. It is often a blend of traditional design and new emerging technology. The new technology is incorporated because it is identified as having some potential improvement in airframe weight, performance, survivability, supportability, or cost.

This evolution and technology insertion process will be discussed in terms of the key subareas of structural concepts and materials, design criteria, and analysis and design techniques.

Structural Concepts and Materials

The evolution of aircraft structures has been an interesting transition.¹ Structures have evolved from the aluminum alloy, stressed skin, semimonocoque construction of the C-47 (DC-3) in the 1930s to the 37% composite material, dual structural load path flying wing of the B-2, and the 42% composite V-22 in the 1990s. The cliché “what goes around comes around” describes this evolution best.

Hoff² traces semimonocoque composite (wood) fuselage structure in the United States to the Vega, built by Lockheed in 1927. The Vega aircraft structure was based on U.S. Patent number 1,425,113 issued to Malcolm and Allan Loughhead (Lockheed), J. Northrop, and T. Stadlman, with Northrop as the principal designer.

The design consisted of strips of spruce glued together in a concrete mold with a pressurized rubber bag. The middle layer ran circumferentially, and the two outer layers ran lengthwise. The structure was said to be so stiff that only moderately sized frames or semimonocoque were required. Figure 1 shows a Vega semimonocoque wood fuselage.

In production from 1927 to 1935, the Vega made history. In 1930, Wiley Post won the Los Angeles to Chicago National Air Race and in 1933 made the first around-the-world solo flight in one. In 1932, Amelia Earhart became the first woman to fly the Atlantic alone, also in a Vega.

The monocoque (derived from the Greek word monos, meaning single, and the French word coque, meaning shell²) fuselage is actually attributed to a Frenchman, Louis Béchereau, who built the fuselage in 1912 for the Le Monocoque Deperdussin monoplane by gluing on a mold three layers of tulip wood. One layer ran fore and aft, the second in a right-hand spiral, and the third in a left-hand spiral.

According to Gibbs-Smith,³ a Swiss named Ruchonnet designed the first wood monocoque structure in 1911 although it was not applied until the Deperdussin of 1912.

Coleman⁴ indicates that Northrop first adopted the molded plywood fuselage approach for the Northrop designed Loughhead Aircraft Corporation model S-1 sportplane in 1918. This was after Stadlman (Loughhead's construction foreman) studied a German Albatros Company model D. Va biplane. The Albatros Company had been manufacturing three-ply, wood, monocoque fuselages as early as 1916.

Wood continued to be used, off and on, up to the nonstrategic material spruce and balsa sandwich structure of the Mosquitos produced by de Havilland during World War II. De Havilland continued to produce wood fuselages into the postwar years.

The first all-metal airplane, an integrally braced monoplane, was designed and constructed by Hugo Junkers in 1915. Junkers obtained a Deutsches Reich patent for an all-metal flying wing in 1910. In Ref. 5, Junkers indicates that the wing for his initial sheet iron airplane prototype was a “supporting-cover” concept, where all tensile, compressive, and shearing forces are taken up by the wing cover. Because the welded iron wing weight resulted in a poor airplane rate of climb, he changed the wing design in subsequent models, to “react bending moments with a more efficient frame or trestle of tubes.”

The first smooth skin metal monocoque fuselage, patent number 1,557,855 was issued to Flavius E. Loudy in 1921 (see Fig. 1 from Ref. 6) by 1931.

In 1930, Northrop introduced the use of riveted aluminum for the Alpha aircraft. The fuselage was flush-riveted aluminum semimonocoque, and the wing box was aluminum-stiffened skin riveted to flanged shear webs. Northrop's riveted aluminum design used the wing covers to react bending, compression, and tension loads. There were no spar caps per se (see Fig. 2a). This type of structure was used for the Northrop Delta and Gamma aircraft and with slight modification, for the Douglas for the DC-1, DC-2, and DC-3. Figure 2b shows how the upper cover was stiffened by riveted corrugated aluminum. The concept of stressed-skin wing structure is attributed to Adolph Rohrbach of Rohrbach Metallflugzeugbau, GmbH, in 1920.

By 1931, the aircraft yearbook identified the following four U.S. aircraft as employing stressed skin metal construction: Boeing Monomail, Thaden T-4, Northrop Alpha, and Consolidated Fleetster. (Although the Fleetster had an all-metal monocoque fuselage, the wing was initially constructed of plywood.) By 1932 the aircraft was all metal.⁷

There have been many variations on the stiffened stressed skin/frame concept over the years, but for the most part stiffened skin panels supported by frames and doublers around cutouts remains the most prevalent structural concept. Some of the more noteworthy variations on this theme include postbuckled panel tension field design, sandwich structure, integrally stiffened structure, and unitized construction. Postbuckled panel design simply means the panel is required to carry a higher load than that at which the panel skin buckles.

The original application of postbuckled tension field structure⁹ is attributed to Herbert Wagner, at Metallflugzeugbau, in 1925. Postbuckled structural members have been prominent in many aircraft including the B-52, B-1B, F5E, and AV-8B.

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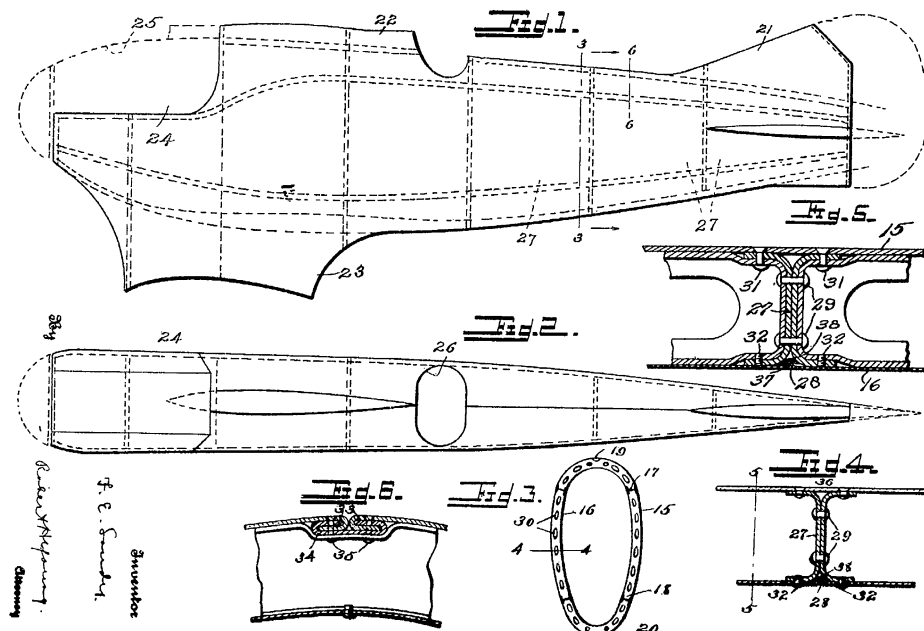


Fig. 1 F. E. Loudy patent 1,557,855, filed 1921.

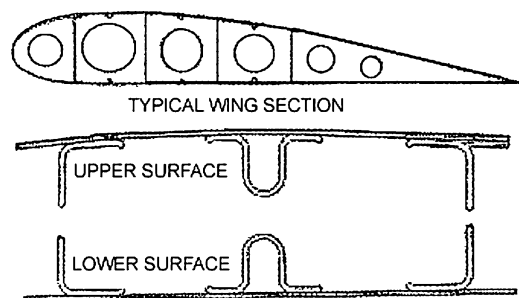


Fig. 2a Northrop wing design.

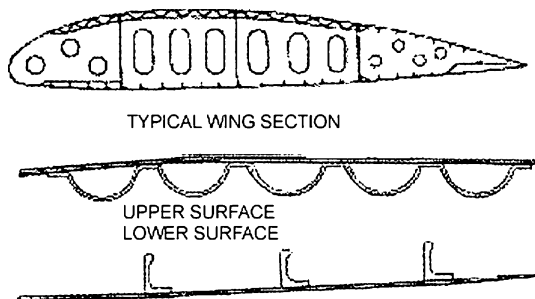


Fig. 2b Douglas D-C2/D-C3/C-47 wing.

For the most part, these concepts have been incorporated into the same basic Northrop skin/frame structural arrangement. One noteworthy exception was the geodetic aircraft construction (called geodesic structure in the United States) used by Vickers Armstrong, Ltd. This type of construction was used for both the 1934 Vickers Wellesley and the 1936 Vickers Wellington. Geodetic construction was described by Thetford¹⁰ as being "immensely strong and could take any amount of punishment from flak and return home again." It was chiefly Duralumin metal lattice with a fabric covering (see Figs. 3a and 3b).

Duralumin (from the French word *dur* meaning hard) was patented by Alfred Wilm in Germany in 1908 and was first produced in 1917. 17s was Alcoa's version of Duralumin. In 1944, Alcoa developed 75S (Al-Zn-Mg-Cu) expressly to meet the aircraft industry's need for higher strength. This alloy first saw service on the B-29 bomber, and according to the 1945 aircraft yearbook,



Fig. 3a Vickers geodetic fuselage production.

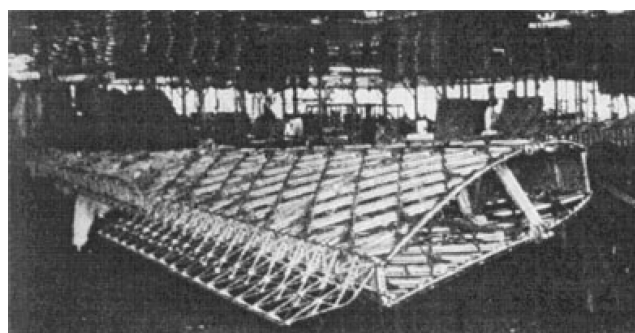


Fig. 3b Vickers geodetic wing construction.

"Practically all the new war planes were utilizing high strength 75S." However, the principal aluminum alloy since 1945 has been Alcoa's 24S (Al-Cu-Mg-Mn) that contained the same alloying elements as the older 17S but in different proportions for greater strength.

Other countries were also quick to adopt Duralumin monocoque fuselage construction. In England, the Short Brothers company built the metal Silver Streak biplane in 1919, which was all Duralumin except for the wing spars, which were steel. The wing consisted of round steel tube spars and Duralumin ribs, with right angle flanges, to which riveted Duralumin covers were joined.

Duralumin metal concept described as Geodetic construction was defined in the 1938 "British Empire Air Annual as an unbroken diagonal system in which torsion loads are taken by continuous members

in tension and compression. When reacting to wing torsion loads, one diagonal system of bracing members is under compression while the other is under tension. Because they are attached at their crossing points, they mutually restrain each other against deflection. Normal spars react bending loads.

Although not adopted, composite geodesic wing and fuselage designs were evaluated by Goldsworthy Engineering for the Beechcraft Starship in the early 1980s. Geodesically stiffened filament composite panels are presently under assessment. A limited amount of test data¹¹ indicates such construction to be very damage tolerant of low-velocity impacts.

However, due to the success of the DC-3, Riveted Aluminum, stiffened skin stringer/frame structure became introduced as the preferred design concept. Howe's comparison of the DC-3 to Abraham Lincoln in "Queen of the Transports"⁸ is just one of many tributes to the dependable and versatile C-47 Skytrain version of the DC-3: "A plain honest background; a simple humble beginning; careful tempering and testing in early life; capability and performance beyond the dreams of parents." Douglas DC-3/C-47 production from 1935 to 1946 totaled 10,654 aircraft. It is estimated that 1000 are still flying today.

Sandwich construction replaces skin stiffeners with lightweight, honeycomb core and fasteners with adhesive bonding. This type of construction permits the use of very thin airframe skin operating at high stress levels without buckling.

The structural efficiency of sandwich construction (lightweight and improved compression stability) has been exploited for many years in aircraft construction. Hoff and Mautner¹² identify some early applications of sandwich construction in airframes. For example in 1924 von Kármán and P. Stock were granted a German patent for a sandwich glider fuselage and that a plywood-cork sandwich wing monoplane was displayed at the French Salon d'Aeronautique in 1938. They indicate that the incentive for sandwich construction was the desire to build a true monocoque airplane.

There is some sandwich structure on virtually every aircraft. The de Havilland Mosquito bomber, of World War II fame, employed bonded wood sandwich structure for wing panels. Wing skins consisted of discrete blocks of spruce core and cedar plywood face sheets. The fuselage was a sandwich of spruce veneer on balsa core with solid spruce core substituted where attachments (concentrated loads) existed.

Although described by Rhodes¹³ as being "robust to battle damage," maintenance problems arose with the bonded airframe in the South Pacific. Tropical organisms and humidity were said to have done more damage than the Japanese. (De Havilland first used a plywood-balsa sandwich for the Albatross fuselage as a four-engined airliner of 1938.)

The more recent 1964 B-70 was constructed of brazed steel honeycomb sandwich [2,043 m² (22,000 ft²) and 68% of the airframe weight]. The C-5 contains 3,250 m² (35,000 ft²) of bonded sandwich, including wing leading edges, slats and wing tips, trailing-edge surfaces, flaps and nacelles, troop deck floor, pylons and fairings, and main landing-gear pod.

The General Dynamics B-58, which broke 12 world speed records, was called the bonded bomber because of the extensive use of bonded aluminum sandwich construction. Developed in the 1950s, aluminum (2024-T86/7075-T6) sandwich panels covered 90% of the wings and 80% of the total airframe. The entire outer skin of the wing was made up of honeycomb core sandwich, and the fuselage was a combination of beaded and honeycomb sandwich panels. Partly because of extremely lightweight sandwich design and partly because of a large bomb/fuel pod that comprised the lower half of the fuselage, the structure made up only 16.5% of the takeoff gross weight and only 14% of the maximum gross weight achieved later in flight refueling.

This lightweight airframe design brought with it significant structural maintenance problems. The Strategic Air Command once estimated that the cost of operating two wings of supersonic B-58 aircraft equaled that of six wings of B-52s.

Of course, this was not all airframe maintenance. The B-58 avionics system consisted of 5000 vacuum tubes and transistors. The thin-

sheet sandwich skin was, however, susceptible to in-service damage caused by handling, walking, and punctures and required periodic time-consuming costly inspections.

According to Petrushka,¹⁴ General Dynamics Director of Structures in 1985, if the B-58 had not been retired early, "it is fair to conjecture that the extreme maintenance problems with honeycomb sandwich could well have become the limiting life factor."

In its many years of using aluminum materials, the aircraft industry has learned several important lessons. The KC-135 aircraft¹⁵ illustrates one example. In 1954, the Boeing 367-80 became America's first jet transport prototype. The KC-135 was a derivative of the Boeing 367-80 (Dash 80), as was the 707, but the lower wing skin of the KC-135 was initially 7178-T6 Al. A small but critical crack length for this material at the KC-135 wing-limit load level led to in-service rapid fracture in several instances.

The concern about loss of an aircraft from degradation of fail safety from widespread fatigue cracking motivated the U.S. Air Force to make a costly modification. The lower wing skins were replaced with 2024-T3 aluminum.

In 1982, 14 years of service experience with the C-5A resulted in greater emphasis being placed on fracture toughness, corrosion resistance, and durability in selecting materials for the C-5B; for example, 7075-T6 fuselage skin and cargo floor skins were changed to 7475-T761 where corrosion was a problem. The wing plank material was changed from 7075-T6 to 7175-T73 for increased toughness.

It was predicted as early as 1985 (Ref. 16) that "lithium containing aluminum alloys (Al/Li) would find significant use in both military and civil aircraft" and that the "ultimate level of utilization of carbon fiber reinforced polymer matrix composites may be somewhat less than predicted, in view of the potential of Al/Li alloys."

The realization set in that Al/Li was not ready to be a wide-scale direct substitute for conventional aluminum aerospace alloys, although it was being used in some production airframe applications in the United States and Europe. At present, it is the high-strength aluminum alloys such as 7150 and 7055 with increased compression strength and balanced fracture toughness and corrosion resistance that are enabling aluminum to maintain its high level of use in the aerospace industry.

However, the use of other metals has increased over time. The first major structural application of titanium for example was the Douglas X-3 that used 629 lb of Rem-Cru, Inc., titanium. This was a commercially pure material intended for the aft fuselage boom and stabilator portions of the aircraft.

The X-3 first flew in 1952, and O. A. Wheelon, a production design engineer with the Douglas Aircraft Company, was awarded the Wright brothers medal for a published analysis of the use of titanium in aircraft construction, entitled "Design Methods and Manufacturing Techniques with Titanium."

One of the first airframe production applications was the F-86 Sabre Jet, which is noted for a kill/loss ratio of at least 4:1 in the Korean War. This aircraft, which flew as the XP-86 in 1947, contained 600 lb of titanium in the aft fuselage and engine areas (Fig. 4).

The history and early use of titanium was solidly wedded to the aerospace industry, and the annealed alpha-beta alloy Ti 6Al-4V has been the workhorse of the industry. This alloy has been used on aircraft from the B-52 to the F-22. This specific alloy composition was first melted and evaluated in 1953 (Ref. 17), at IIT Research Institute, under a U.S. Air Force contract. The Mallory Sharon Titanium Corporation version of this alloy (MST 6Al-4V) was selected shortly thereafter (1954) for use in Pratt and Whitney's J-57 turbojet engine (disks, blades, and other parts) for the B-52.

The first commercial production of titanium, named after the titans of mythology, began in 1946. Titanium was pushed into commercial production as a structural material, in an unprecedented short amount of time by the combined efforts of industry and government to meet military requirements.

Figure 5 shows how rapidly titanium sponge (i.e., refined metal before melting into usable form) production was scaled up. In 1956, 90% of the titanium market was dedicated to military aircraft production. By 1980, less than 20% of aerospace grade was used for

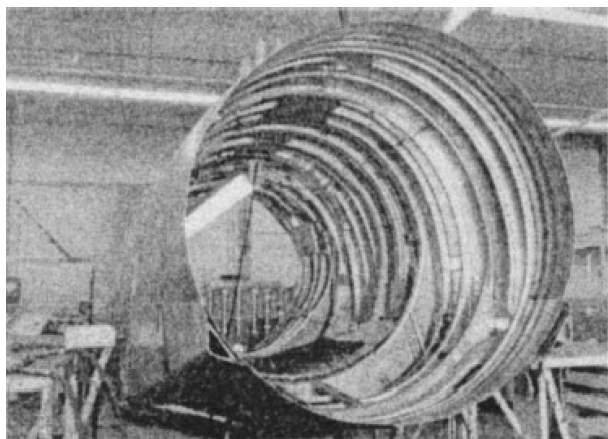


Fig. 4 Titanium aft fuselage in production at North American Aviation in 1954.

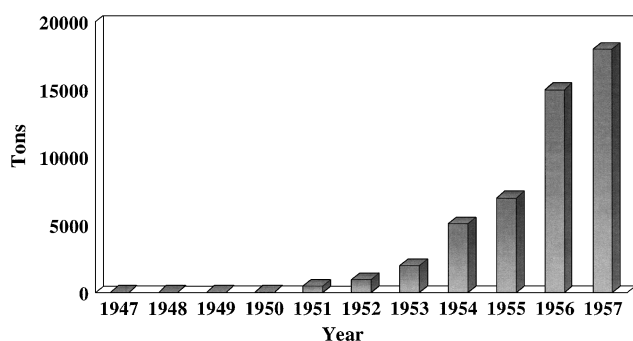


Fig. 5 U.S. production of titanium sponge.

military aircraft. The majority of titanium sponge is processed into titanium dioxide pigment for use as filler in paint, paper, plastics, and rubber.

The most famous U.S. titanium airplane, the YF-12A/SR-71, first flew as the A-12 in 1962 and was fabricated from primarily beta-120 VCA alloy (Ti-13V-11Cr-3Al).

Johnson¹⁸ stated that “of the first 6000 B-120 pieces fabricated, we lost 95%.” The material was described as being so brittle that if it fell off your desk it would shatter on the floor.

The most widely used alloy is Ti-6Al-4V. This alloy is particularly amenable to fabrication by superplastic-forming/diffusion-bonding (SPF/DB) processes. It was, therefore, the subject of a systematic well-planned research and development thrust by the U.S. Air Force and U.S. Navy over a 15-year period from 1970 to 1985.

In SPF/DD takeoffs, the authors¹⁹ indicate that the use of SPF/DB titanium for the aft fuselage of the F-15E resulted in 726 fewer components and 10,000 fewer fasteners. It also achieved 15% weight savings over the C/D models (see Figs. 6a and 6b).

In December 1969 (Ref. 20) a lecture to the Royal Aeronautical Society of England titled, “The History of Metal Aircraft Construction,” was given by Marcus Langley. In the prologue to this lecture, he indicated that his lecture was in some way an obituary of metal aircraft construction for the following reason: “Although, we shall still have metal aircraft for many years to come, quite new materials are beginning to appear and they have as many advantages over metal as metal had over wood. I am referring to such materials as carbon fibers used in matrices of synthetic resins.”

Hindsight now shows that this pronouncement and many similar since then were premature. The airframe of today’s aircraft, 25 years later, is a hybrid of metal and composites. A review of the current status of airframe materials usage (Table 1) shows a fairly equal balance in the use of aluminum, titanium, and composites. This balance is a modern development, driven primarily by performance requirements and affordability.

Production in the United States of high-modulus fiber-composite structures, commonly referred to as advanced composites, began

Table 1 Materials use as a percentage of structural weight

Material	B-2	F/A-18E/F	F-22	V-22	C-17	AV-8B
Composite, %	37	22	25	42	8	26
Aluminum, %	27	27	16	—	70	41
Titanium, %	23	23	37	—	9	—

Table 2 Advanced composite technology programs

Agency	Acronym	Program
U.S. Army	ACAP	Advanced composite airframe program
NASA	ACEE	Aircraft energy efficiency
U.S. Air Force	DMLCC	Design and manufacture of low-cost composites
NASA	ACT	Advanced composites technology
U.S. Navy	AV-8B	Composite structure development program
ARPA ^a	ACP	Affordable composites for propulsion
DARPA ^b	APMC	Affordable polymer matrix composites
DOD ^c /industry	CAI	Composites affordability initiative

^aAdvanced Research Projects Agency.

^bDefense Advanced Research Projects Agency.

^cU.S. Department of Defense.

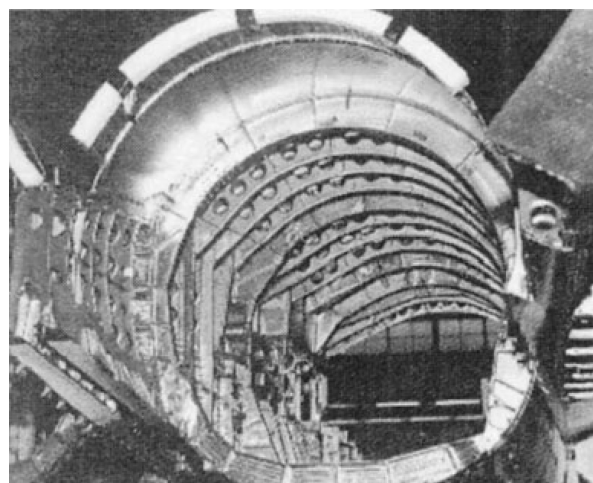


Fig. 6a F-16C/D conventional aft fuselage.

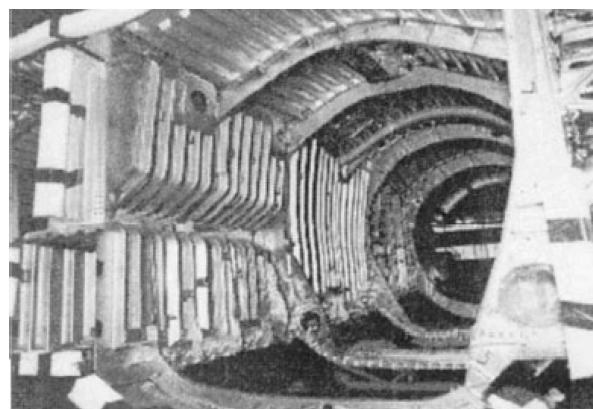


Fig. 6b F-15E SPF/DB aft fuselage.

with boron epoxy F-14 and F-15 horizontal stabilizers in the early 1970s. The technology was fostered by the U.S. Air Force Boron Filaments and Composites Advanced Development Program initiated in 1964 as a result of a Project Forecast recommendation. Experimental carbon-epoxy structures were flying in 1970, and carbon epoxy came to the forefront as the most cost-effective advanced composite material in the mid-1970s with production applications on the F-15, B-1, and F-16. In 1978, carbon epoxy was accepted by the Navy for F-18 and AV-8B primary wing box structures.

Table 3 Composite structure weight savings

Component	Baseline weight, lb	Composite weight, lb	Weight savings, %
<i>Flight service components</i>			
Empennage			
F-111 horizontal stabilizer	1142	866	24
F-14 horizontal stabilizer	1005	825	18
F-15 horizontal stabilizer	527	395	25
F-5 vertical stabilizer	119	83	30
DC-10 vertical stabilizer	1005	834	17
L-1011 vertical stabilizer	858	642	25
B-737 horizontal stabilizer	262	204	22
Wing			
AV-8B wing	1143	949	17
F-18 wing	1843	1641	11
A-6 wing (selective reinforcement)	6392	6400	0 ^a
C-130 wing box	4800	4200	12.5
Fuselage			
UH-60 rear fuselage	444	399	10
AV-8B forward fuselage (selective reinforcement)	229	171	25
CH-53 Aft fuselage	1159	947	18
B-1 dorsal longeron	1485	829	44
Other			
C-5 slat	241	190	21
B-727 elevator	131	98	25
DC-10 rudder	91	67	26
LO-1011 aileron	140	107	23
F-15 speed brake	112	89	21
B-1 weapons bay door	147	129	12
F-5 flap	34	25	26
B-1 flap	87	73	16
B-1 slat	74	61	18
F-4 rudder	64	42	34
F-11 spoiler	20	17	15
Space shuttle doors	4150	3196	23
DC-9 nose cowl	24	13	46
F-16 landing gear door	53	42	20
A-7 speed brake	123	74	40
<i>Research and design demonstration components</i>			
Wing			
F-15 wing	2155	1787	17
F-100 wing skin	616	475	23
Fuselage			
F-111 fuselage	626	510	18
CH-54 tail cone	464	397	14
YF-16 fuselage	572	451	21
F-5 fuselage	914	677	26
Other			
B-1 vertical stabilizer	658	539	18
B-1 horizontal stabilizer	3315	2844	14

^aLoad capability 18% increase.

Table 3 is a sample of some of the early composite structure demonstrations. Table 3 consists of a list of composite structures for which a metal baseline weight was available.

There were of course many other composite structures, some of which did not have a metal baseline or for which a metal counterpart would not even be practical. Some of those that come to mind are the Rockwell highly maneuverable advanced technology remotely piloted research vehicle and the Grumman X-29 forward-swept-wing demonstrator. Both of these designs exploited the advantageous aeroelastic characteristic of flexible composite wings.

The bulk of advanced composite material being used in flight in the United States is fabricated from unidirectional tape. The tape consist of AS-4 or IM7 carbon fiber in 3501 thermoset epoxy matrix preimpregnated.

Although developed some time ago, the Slingsby T67 Firefly was selected in 1992 for the U.S. Air Force enhanced flight screener (T3A). The basic airframe started as a wooden construction that Slingsby redesigned as primarily a wet layup, glass fiber, polymer composite airframe.

Fiberglass-reinforced polymer construction has been used for many years in the glider industry. It has been argued that the Horten brothers of Germany created the world's first composite aircraft, the Ho V, which began as a glider in 1936. The fuselage and wings for the first Ho V were fabric-covered D-tubes molded from paper-filled phenol resin. R. Horten noted that such molded plastic construction promised to reduce production time and cost.

The term "molded reinforced plastic plane" was being used by 1941 (Ref. 21). Although nearly every form of material was tried as filler, resin-coated wood veneer strips and sheets (usually mahogany or spruce) were the most popular reinforcing agents. Impregnation by liquid phenolic- or urea-type resins was common.

Both Japan and Germany built highly successful fiberglass-composite soaring planes in the 1950s. Several of these are described in Ref. 2.

The 42% by weight composite structure, Bell/Boeing Osprey V-22 first flew in 1989. The critical link between the blade and the rotor hub (the yoke) of the V-22 is a composite structure. The yoke carries the blade centrifugal and lifting forces, transmits engine torque, and permits lead-lag and flapping motion. Also, each yoke arm can accommodate pitch changes.

Helicopter rotor-blade designers established an early position at the forefront of the development and application of advanced composite structures and continue to design to maximize the benefits of the rotor blade. A Vertol CH-47 boron/epoxy aft rotor blade was flight tested in 1969. Of course, glass fiber-reinforced blades were in development and even production several years before that.

Although weight reduction in helicopters is important, it has been the adaptability of the composite to complex aerodynamic shapes that has enabled them to play a major role in helicopter structures. The concepts of hingeless and bearingless rotors were made practical by the design flexibility of composites. Composite blades achieve virtually unlimited fatigue life.

Most composite applications came about as a result of improved performance at an acceptable cost. By the mid-1980s, the structural efficiency of composites was fairly well recognized throughout the aerospace industry. Emphasis then shifted to lowering development and production costs of composite assemblies. The goal was to achieve a lower cost than that of the competing aluminum construction. Expanded use will require achieving this goal, as well as projected long-term durability and supportability benefits.

The emphasis on design to cost will continue to be facilitated by integrated product teams and computer-aided design tools. How well composite producibility and supportability issues are addressed in the future will determine the extent and the rate at which composite airframe applications will expand.

Years of experience with composite materials have also resulted in lessons learned. For example, delaminations were found in the wing skins and substructure of U.S. Marine Corps AV-8B and RAF UK-58 Harriers. Investigations revealed that these delaminations were caused by gaps between the skin and substructure that were not properly shimmed prior to fastener torqueing the assembly of the wings. To correct this problem, and prevent its occurrence in other composite aircraft production, the U.S. Navy now requires that liquid shimming be used to fill gaps between mating surfaces in composite structures. Additionally, closer dimensional tolerances are placed on mating surfaces to ensure a better fit.

In their *Journal of Aircraft* paper, Logan and Soudac²² maintain that a primary motivation for developing bonded honeycomb structure was to significantly reduce the fabrication costs of mechanically fastened construction by significantly decreasing the number of parts associated with the method.

Some recent record-setting aircraft that used composite sandwich airframes include the Hexcel Voyager and the Boeing Condor. In 1987, Richard Rutan and Jeana Yeager completed their nine-day, nonstop, unrefueled 40,200 km (25,000-mile) flight around the world in the graphite composite Hexcel honeycomb sandwich voyager. The Voyager's takeoff weight was more than 10 times the structural weight.

The high-altitude, Boeing-built, uncrewed graphite/Kelvar® epoxy/Nomex honeycomb Condor has a 61 m (200-ft) wing span. It has a 36.7 aspect ratio and weighs only 96 N/m² (2 lb/ft²), which enabled it to set an altitude record of 20,405 m (66,980 ft) (for piston-powered aircraft) and to stay aloft for two days in 1989.

Additionally programs have been initiated to foster the development and transition of composites (see Table 2). Although these initiatives were seldom adequately funded to complete all initially planned objectives, they nonetheless were able to sustain substantial funding for some limited period of time.

Minimizing fastener installation time and cost has been a goal of manufacturing engineers ever since the first all-metal airframes. Barbe²³ describes how the Curtis–Wright Corporation explored stitching of metal aircraft parts during World War II to reduce aircraft fabrication time associated with riveting and spotwelding. An automotive industry metal stitcher (Fig. 7) was adapted for airframes to do metal-to-metal and dissimilar material joining. Stitching and weaving is now being employed to create three-dimensional carbon-fiber preforms. Such a stitching machine²⁴ is funded by the NASA Advanced Composites Technology program (see Fig. 8). This machine, developed by Ingersoll Milling Machine and Pathe technologies, has four high-speed stitching heads capable of 800 stitches/min.

Fastener minimization has also been accomplished by the use of large forgings and castings, which have facilitated the creation

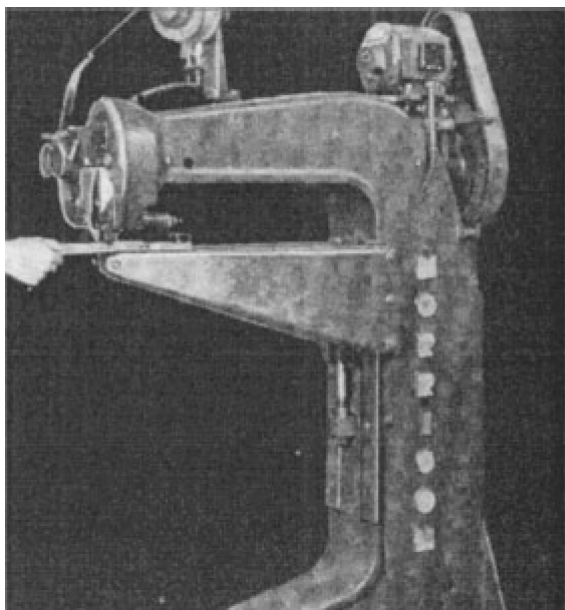


Fig. 7 Curtis–Wright metal stitching machine.

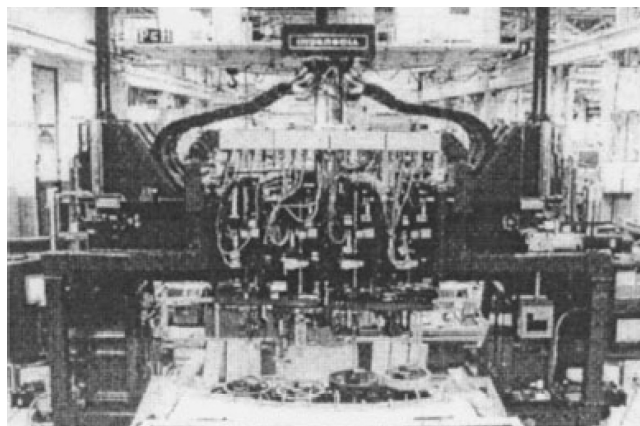


Fig. 8 Composite material stitching machine.

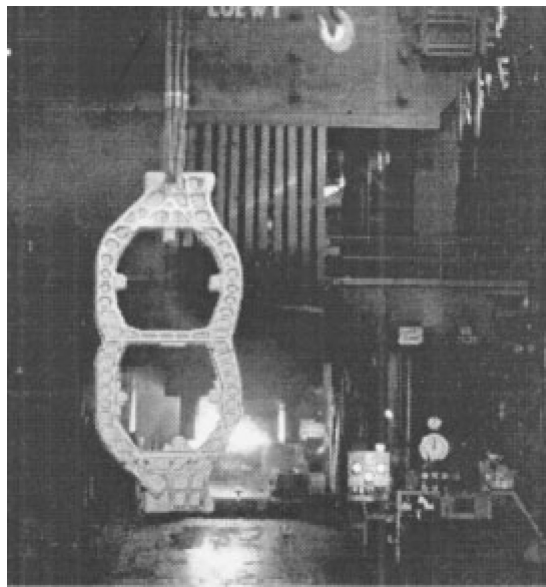


Fig. 9 F-22 aft-fuselage frame forging produced on Wyman–Gordon 50,000-ton hydraulic press.

of one-piece substructures. Figure 9 shows a Ti-6Al-4V F-22 aft-fuselage frame produced by Wyman–Gordon. The F-22 wing carry-through bulkhead is fabricated from the largest Titanium forging, by surface area to date [9 m² (96 ft²), 2,975 kg (6560 lb)]. Four bulkheads in the midfuselage are made of titanium. The forward and aft boom structure on either side of the aircraft's empennage section are formed from integrally stiffened titanium isogrid panels that are electron beam welded to forged titanium frames. The wing attach fittings and rudder actuator support are hot isostatically pressed (HIP) titanium castings.

The A-6E composite wing rear spar is a 8.2 m (27-ft) titanium forging, and large one-piece titanium bulkhead forgings are used by the F/A-18 E/F aircraft.

The Bell-Boeing V-22 Osprey engineering and manufacturing development aircraft (first flight 5 February 1997) used a one-piece titanium casting for the transmission adapter case stow ring that was previously 40 separate riveted components. This part serves as the primary support structure for the rotor system, propotor gearbox, and engine and nacelle components. The Howmet Corporation indicates that a premium-quality consistently homogenous component is achieved by the HIP processing so that allowables do not have to be reduced by conventional casting factors.

Recently RMI Titanium of Ohio developed a stronger, more damage-tolerant alloy (Ti-6-22-22S) that is finding application on the F-22 joint strike fighter. Another notable alloy was developed by Timet Corporation. Their alloy 10-2-3 is being used in the main landing gear and Timetal 21S for the nacelle cowl, plug nozzle, and Pratt and Whitney Aircraft engine components of the Boeing 777 aircraft. Titanium usage on Boeing aircraft has increased from 2% empty weight of the 737 to 8% of the 777. The McDonnell Douglas C-17 transport also utilizes alloy 10-2-3 for landing-gear components.

Noting this trend toward increased titanium use, a 1991 *Advanced Materials and Processing* magazine article was titled "Titanium Staves Off Composites."²⁵ In reality, titanium has grown in use partly because of its compatibility with composites both from a thermal expansion characteristic and resistance to galvanic corrosion when in contact with carbon epoxy.

The generation of composite structures technology applied to the B-2, F/A-18E/F, V-22, and RAH-66 has been in development for 30 years. Its level of maturity is a result of a continued investment by the research and development structures and materials communities of all three services and NASA. No other class of material has received such high level of funding support for such a long time. Numerous composite structures built from the 1960s to 1980s

verified composite structure advantages (primarily weight savings) leading to more extensive application of composite airframes in the 1990s when affordability became the major issue.

Design Criteria

Up to the mid-1950s, the method for ensuring structural integrity was essentially the same one that the Wright brothers used in 1903. A stress analysis was substantiated by a static test to a load level greater than that expected in flight.

The method consists of applying a factor of safety to the flight loads and verifying that the structure could sustain this load by testing a full-scale airframe. The factor of safety was developed to account for the following sources of variability²⁶: 1) uncertainties in loads, 2) inaccuracies in structural analysis, 3) variations in material properties, 4) deterioration during service life, and 5) variations in construction quality.

The aircraft yearbook in 1921 states: "The fatigue of metals or progressive failure from incipient cracking is the greatest bugbear of the designer." The Bureau of Standards was reporting flexural fatigue data on duralumin sheets as early as 1922. However, Hoff² indicates that the first mention of fatigue in government airworthiness regulations did not appear until 1945. The concept of metal fatigue goes back a long way to the failures of railway cars and locomotive axles in the 1840s. Shultz²⁷ credits Wöhler of Germany in 1860 with the concept of designing for a finite fatigue life. In 1853, French engineers were inspecting axles of horse-drawn mail coaches for cracks and recommending their replacement after 60,000 km. However, Jenkin²⁸ notes that it was not until 1914 that sufficient test data were available to consider incorporating fatigue limits into aerospace structural steel specifications. In 1918, the first full-scale fatigue test of a large aircraft component was carried out in the United Kingdom at the Royal Aircraft Establishment.

Many papers were written in the 1940s predicting increases in airframe fatigue failures. Bland and Sandorff²⁹ forecasted that increases in fatigue troubles would stem from the following trends that existed in 1943: 1) a drive toward higher cruising speeds, 2) the continuous increase in the ratio of useful load to gross weight, and 3) the increase in wing loading, gross weight, and service life.

Bland and Sandorff provide stress-number-of-cycles curves on the 17S and recommend designing on the basis of life expectancy. The authors also recommend determining service loadings from statistical flight data recorded on existing aircraft but cautioned that such data are applicable only to airplanes of speed, wing loading, size, and mission similar to those in which the data were obtained.

Schütz²⁶ indicates like Gassner of Germany had warned, as early as 1941, that by taking advantage of the higher static strength of the new aluminum alloy (7000-type) a shorter fatigue life would result. The research goal of that time was to obtain sufficient data to be able to reduce factors of safety that resulted from ignorance of the strength of materials.

An International Committee on Aeronautical Fatigue was founded in 1951 at the suggestion of Plantema of The Netherlands. It was two U.K. Comet crashes in 1954 and a series of U.S. B-47 catastrophic accidents in 1958, however, that focused U.S. Air Force attention on structural fatigue and led to The U.S. Air Force Aircraft Structural Integrity Program (ASIP). This program adopted a fatigue methodology approach commonly referred to as safe-life. This approach was used in the development of U.S. Air Force aircraft in 1960s, but was not adequate from the standpoint of addressing manufacturing and service-induced damage. Consequently, aircraft designed both prior to and subsequent to the safe-life method continued to experience premature failures. The U.S. Air Force ASIP was revised to include the incorporation of a damage tolerance/durability philosophy into the design process. Although initially applied to B-1A and modifications of the F-111, C-5A, and F-4, the damage tolerance/durability requirements were formally established in MIL-STD-1530A in 1975.

The U.S. Navy ASIP is similar. It requires fatigue life of the structure to be designed such that failure of a full-scale test article

Table 4 Airframe design development tests

Aircraft	Elements tested
B-2	160,000 nonmetals, 1,450 metallic, 6 leading edge 74 wing panels, 40 engine exhaust panels, 27 joints
C-17	6,000 coupons, 400 components
F-22	Allowables 14,500 composites, 5,300 metals Structural integrity 4,100 metals, 8,800 composites Reproducibility 80 full and subscale composite, 15 titanium structures
F/A-18 E/F	10,000 materials characterization/processing tests, 165 elements and components
V-22	9,700 coupons, 550 elements of components

will not occur and that the structure will be free from any defects such as cracks, deformation, loss of modulus, disbonding, or fastener hole deformation and delamination when subjected to a minimum of two lifetimes.

Spurred on by joint service airframe requirements of the Joint Advanced Strike Technology program, the services have developed a joint service guide specification for aircraft structures.

All U.S. Air Force aircraft since the F-16 have been subject to durability/damage tolerance requirements. Each of the tests required by ASIP has served to identify deficiencies in structural design and analysis, resulting in modifications to the structure that provided a safer and more supportable airframe.

The full-scale airframe static, fatigue-durability/damage-tolerance tests are expensive, and therefore, there should be a reasonable expectation that they can, with minor design modifications, be accomplished successfully. This generally means conducting an extensive structural element and subcomponent test program in support of the ASIP. These tests shown in Table 4 address all of the critical load paths.

This type of building-block program, based on, testing large numbers of coupons, elements, and components prior to full-scale hardware has evolved as the risk-reduction approach for new system airframe development.

Advanced technology demonstrators (ATD) introduce technology through prototyping, thus accelerating transition of new technologies. Failure in an ATD program is more accepted as a lesson learned rather than a failure of the contractor and procuring agency.

The data presented in Table 4 suggests that composite structures require more extensive design and certification testing than do metals. The additional testing requirements add cost to the design allowables and design certification efforts. This was clearly identified as a lesson learned by Vosteen and Hadlock in November 1994.

In the case of composite structural assemblies, however, when the structure satisfies all static load conditions, fatigue is generally not a problem. The effects of moisture and temperature on design margins can be established at the coupon and subcomponent level with the full-scale component margins adjusted appropriately.

The one thing that described programs had in common was a substantial finite element analysis effort (both global and local detail models) to support test hardware design and to certify the airframe for adequate design margins. These programs demonstrate the importance of a thorough analysis effort coupled with a complete critical area design development test program to validate the design.

Analysis and Design Techniques

The introduction of digital computers in the 1950s paved the way for emerging multidisciplinary design technologies applicable to the integrated design of a variety of engineering products including air and space vehicles. This section is a brief outline of the developments that led to such software systems as ASKA, STARDYNE, NASTRAN, ANSYS, ABAQUS, and ASTROS.

Before discussing the chronology and significance of structural analysis developments, it is appropriate to distinguish between two levels of analysis: 1) analysis of structural elements and 2) analysis of built-up structures.

Analysis of Structural Elements

Structural elements or components can be categorized into three types: line elements, surface elements, and solids. The analysis of such elements has been developed and documented over the last 150 years by Euler, Lagrange, St. Venant, and others. Todhunter and Peterson³⁰ and Timoshenko³¹ document the chronology of these developments. Over a dozen solid mechanics books by Timoshenko constitute a rich compendium of analysis methods for structural elements. The range and applicability of this continuum approach (the solution of differential equations) is, however, limited to very restricted cases of loading, boundary conditions, and element shapes. Such methods as Rayleigh–Ritz, Galerkin, and finite differences are approximations. These methods replace a continuum model by a discretized equivalent. The result is a set of algebraic equations that can be solved by digital computers.

Analysis of Built-Up Structures

Built-up structures are simply constructed out of many structural elements joined together at their boundaries. The differential equation approach for the solution of built-up structures is intractable. As an alternative, a number of discretized procedures have been developed since the early 1900s. A structure is considered determinate if its internal forces can be solved by the force and moment equilibrium equations alone. Otherwise, it is considered an indeterminate system. Analyzing complex indeterminate systems was a daunting task before the advent of digital computers.

The force and displacement methods that emerged in the 1950s were a major breakthrough in structural analysis. It refers to the discretized model of the structure in which the continuum is replaced by a finite number of grid points. Each grid point is assumed to have six degrees of freedom (DOF) in space. These DOF are forces and displacements. In the force method, the unknowns are the set of forces released to make the indeterminate structure determinate. Enforcement of compatibility conditions results in force-displacement relations connected by a flexibility matrix. In the displacement method, the unknowns are the displacements of the grid points, and the applied forces are the known quantities. The two are connected by the stiffness matrix of the structure. Collectively the stiffness and flexibility (displacement and force) methods are labeled as matrix structural analysis. Argyris and Kelsey³² provided a comprehensive outline of these methods in 1954. Matrix structural analysis developed along two distinct paths: force method and displacement method. Each has advantages and drawbacks in its application to complex structures.

The publication of the finite element concept by Turner et al.³³ in 1956 is a significant event in the development of automated structural analysis. The basic premise of the finite element formulation is to divide the continuum into small domains and view the total system as a summation of the small domains, called the finite elements. The stiffness of these elements can be generated independently with the total system stiffness matrix assembled by a summation, after the appropriate coordinate transformations. This procedure provided the necessary breakthrough for automated analysis by computers.

The pace of finite element analysis development was accelerated by the three matrix methods conferences sponsored by the U.S. Air Force Flight Dynamics Laboratory (AFFDL) and the Air Force Institute of Technology in 1965, 1968, and 1971 (Refs. 34–36).

In the 1960s, several government laboratories and aerospace companies initiated the development of large-scale structural analysis software. AFFDL sponsored the development of the FORMAT software system at Douglas Aircraft Company and the MAGIC software system at Bell Aerospace Company. FORMAT and MAGIC provided the impetus for the development of general purpose structural analysis software pursued by NASA in 1964.

The combination of the NASA Goddard Research Center leadership, the Computer Sciences Corporation's system software expertise, and MacNeal and Swendler Corporation's solution experience of practical problems produced the software system called NASTRAN. NASTRAN was first released in 1968. The commercial version of NASTRAN was initiated in 1972 under the name of MSC/NASTRAN. It has emerged as truly general-purpose software

by making the model input independent of the details of the mathematical operations internal to the software. It now contains extensive nonlinear-analysis, heat-transfer, aeroelasticity, and thermal-analysis modules. A more complete discussion of this development can be found in Schmidt's paper.³⁷ It in turn was the impetus for the development of such programs as the structural analysis programs series, ANSYS, and ABAQUS.

In 1960, Schmidt³⁷ proposed coupling finite element analysis and mathematical programming methods to accomplish structural design optimization. The optimization problem is stated as a minimization or maximization of an objective function subjected to a set of behavioral and/or manufacturing constraints. Three basic elements of an optimization problem are 1) definition of a performance or objective function, 2) definition of a set of constraints, and 3) identification of the variables that define the functions. The variables define an n -dimensional space in which the search for the optimum can be carried out by various linear and nonlinear programming methods. There are a variety of gradient and nongradient methods available to carry out this search for the optimum. Venkayya³⁸ gives a comprehensive review of these methods and software programs.

The scope of structural optimization expanded into multidisciplinary design optimization (MDO) in the 1980s and 1990s. MDO is defined here as an optimization problem where the constraints and/or objective functions are derived from more than one discipline. In the context of airframe design optimization, the constraints and the objective functions are coming from static and dynamic aeroelasticity, such as lift and control surface effectiveness, flutter, divergence, stresses, displacements, frequencies, and possibly from electromagnetics as well. There are a number of commercial programs available for MDO. ASTROS, ELFINI, LAGRANGE, NASTRAN, and GENESIS are some examples. These software developments also spawned extensive development in pre- and post-processing software. Some examples are PATRAN, IDEAS, and HYPERMESH.

The finite element based computer-aided design tools available today not only improve the design team's productivity, but also these tools facilitate exploration of several design concept options and assessment of the design concept's sensitivity to modifications.

We have evolved from aerospace companies that have their own proprietary finite element programs to commercial finite element software systems. These systems integrate finite element method into multidisciplinary tools capable of automated optimum design in a concurrent engineering environment.

The one area that can be considered as having undergone a revolutionary change in a relatively short period of time is the application of CAD/CAM and computer-aided engineering tools. Automated design tools implemented by integrated product development teams (IDTs) have truly revolutionized the design process.

Concurrent engineering facilitated by computers is providing these design/build teams with the disciplines necessary to design, tool, and manufacture major structural assemblies. Examples of such teams are given in Table 5.

For those who have watched the slide rule replaced by the computer, the drafting table replaced by workstations, blueprints replaced by digital data representations, and physical mockups replaced by electronic models, it has truly been revolutionary. Concurrent engineering implemented by integrated product teams supported by computer-aided design tools has brought to the forefront a more balanced design process. Design teams are able to integrate the sometimes conflicting requirements for increased performance, affordability, maintainability, and survivability.

Table 5 Concurrent engineering teams

Aircraft	Number, IPTs
Lockheed F-22	84 (divided into several tiers)
Boeing/Bell V-22	72
McDonnell/Northrop F/A-18E/F	407
General Electric F414	46 secondary, 8 primary integrated concurrent engineering design/development teams
Pratt and Whitney F-19	100

Conclusions

If airframes for future systems are established at the conceptual level on the basis of historical trends, as has been done for aircraft, then basic approaches to react external loads, load path definition, and adjacent structural member joining will be accomplished in the same manner as the last 60 years (i.e., the riveted skin/stringer/frame concept will be perpetuated.) For example, the 50 m (165-ft) wing span, 265,400 kg (585,000-lb) takeoff gross weight (TOGW), C-17 transport uses the same basic aluminum stressed skin three-spar multirib structure developed for the 1935 DC-3 29 m (95-ft) wing span, 10,900 kg (24,000-lb) TOGW transport.

The significant advancement in structural efficiency required for future, affordable systems will require breaking from the skin/stringer/frame concept mold. Employing new/engineered materials such as advanced composites and advanced alloys in existing structural concepts does not exploit their full potential.

Adaptive structures capable of active load control, compensating for damage, or improving aerodynamic efficiency are facilitated by multifunctional structures and in turn are an enabling technology for adaptive aircraft. The development of multifunctional and adaptive structures exploiting engineered material capabilities should be the forcing function to break from the traditional skin/stringer/frame mold. Engineered materials and tailored adaptive structures will not in the future be buzzwords, but rather enabling technology for more structurally efficient/affordable airframes.

Design synthesis and optimization tools are being developed that can graphically display stress contours, deformations, and mode shapes for many structural arrangements. This results in more efficient structural member placement.

Wilbur Wright, 13 May 1900, noted "It is possible to fly without motors, but not without knowledge and skill. This I conceive to be fortunate, for man, by reason of his greater intellect, can more reasonably hope to equal birds in knowledge, than to equal nature in the perfection of her machinery."

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